# Mechanics

Rotational motion

PHYSICS EXPERIMENT

# Moment of Inertia

## DETERMINE THE MOMENT OF INERTIA FOR VARIOUS TEST BODIES

- Determine the torsional coefficient Dr between for the springs used to couple the objects.
- Determine the moment of inertia J for a dumbbell bar without any added weights
- Determine the moment of inertia J as a function of distance r of a weight from its axis of rotation.
- Determine the moment of inertia *J* for a circular wooden disc, a wooden sphere and both solid and hollow cylinders
- Verify the parallel-axis/Huygens-Steiner theorem

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Fig. 1: Experiment set-up.

## GENERAL PRINCIPLES

**The inertia of a rigid body with respect to a change in its rotational motion around a fixed axis is given by its moment of inertia *J*. This is dependent on the distribution of mass in the body relative to the axis of rotation and increases at greater distance from the axis of rotation itself.**

In general, moment of inertia is defined by means of a volume integral:

$$(1)\quad J = \int_V r_s^2 \cdot \rho(r) \cdot dV$$

$r_s$: component of r perpendicular to the axis of rotation
$\rho(\boldsymbol{r})$: Distribution of mass in the body

Using as an example a dumbbell set, which has two weights of mass *m* symmetrically arranged at a distance *r* from the axis of rotation, then the moment of inertia is as follows:

$$(2)\quad J = J_0 + J_m = J_0 + 2 \cdot m \cdot r^2$$

$J_0$: Moment of inertia of dumbbell bar without weights
$J_m$: Moment of inertia of weights

Now we can attach various test bodies to a twisting axis so that they can oscillate. If the period of oscillation is *T*, then the following is true:

(3) $T = 2\pi \cdot \sqrt{\frac{J}{D_r}}$

$D_r$: Torsional coefficient of coil springs

The means that the period of oscillation $T$ will be greater when the moment of inertia $J$ is larger.

From equation (3) it is possible to obtain a formula for determining the moment of inertia:

(4) $J = D_r \cdot \frac{T^2}{4\pi^2}$

The torsional coefficient of the coil springs can be determined with the help of a spring dynamometer:

(5) $D_r = \frac{F \cdot r}{\alpha}$

α: Deflection from equilibrium state

## LIST OF EQUIPMENT

| | | |
|---|---|---|
| 1 Torsion Axle | U20050 | 1008662 |
| 1 Photo Gate | U11365 | 1000563 |
| 1 Digital Counter | U8533341 | 1001032/3 |
| 1 Barrel Foot, 1000 g | U13265 | 1002834 |
| 1 Tripod Stand 185 mm | U13271 | 1002836 |
| 1 Precision Dynamometer 1 N | U20032 | 1003104 |
| 1 Set of Test Bodies for Torsion Axle | U20051 | 1008663 |

## SET UP AND PROCEDURE

- Set up the experiment as shown in Fig. 1. Align the rotational axis to be horizontal to the stand base with the help of the spirit level and the levelling screws.
- Connect the photo gate to channel A of the digital counter. Set the operating mode selector switch on the digital counter to the symbol for measuring the periods of a pendulum.

**Notes:**

- When deflecting the bar, do so in such a way that the coupling springs are forced together and do not bend outwards.
- At the start of the oscillation, it is recommended that a deflection angle of 180° be used (max. 360°).

**Determining torsional coefficient $D_r$ for coupling springs**

- Suspend the spring dynamometer at distances $r$ = 5, 10, 15, 20, 25 and 30 cm from the centre of the rotational axis in sequence on the dumbbell bar and deflect them by an angle $\alpha$ = 180° = $\pi$. Make sure that the dynamometer hangs down at right angles to the dumbbell bar.
- Read off the force values needed to deflect the bar at the various distances from the dynamometer. Enter all the results into Table 1.

**Determining the moment of inertia $J_0$ for the bar without weights**

- Deflect the dumbbell bar without any weights attached by an angle of 180° and use the digital counter to measure the period of oscillation $T_0$.

**Determination of moment of inertia $J$ as a function of the distance of weights $r$ from the axis of rotation**

- Attach the two weights to the bar symmetrically at distances $r$ = 5, 10, 15, 20, 25 and 30 cm to the left and right of the axis of rotation.
- Do not touch the screws which press the ratchet balls against the dumbbell bar. They are adjusted in such a way that the weights can be moved but can also be secured against centrifugal force.
- Deflect the bar by 180° and use the digital counter to measure the periods of oscillation $T$ for each distance. Enter the results into Table 2.

**Determine the moments of inertia $J$ for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder**

- Attach the sample bodies to the torsion axle one after the other. Use the pan for the solid cylinder and the hollow cylinder.
- In order to measure the length of a period of oscillation, use a suitable method to attach a paper flag to each of the samples in order to break the photo gate beam.
- First deflect the wooden disc by 180° then fit the wooden sphere and do the same. In each case, measure the period of oscillation and enter the results into Table 3. Use the white markings on the samples to help you with the orientation for the deflection.
- Deflect the pan by 180° and measure the period of oscillation. Enter the value into Table 3.
- Put the solid cylinder on the pan, deflect it by 180° and measure the period of oscillation. Do the same for the hollow cylinder and enter all the results into Table 3. Use the white markings on the samples to help you with the orientation for the deflection.

**Verification of Huygen-Steiner theorem**

- Fasten the bolt through the boreholes at distances $a$ = 0, 2, 4, 6, 8, 10, 12 and 14 cm from the centre of the disc
- For each of the various positions of the bolt, mount the disc onto the axis, deflect it by 180° in each case and measure the period of oscillation. Use a suitable method to attach a paper flag to the disc. Enter the values into Table 4.

# SAMPLE MEASUREMENT

**Determining torsional coefficient $D_r$ for coupling springs**

Tab. 1: Measurements of force *F* at a distance *r* from the centre of the axis of rotation when the dumbbell bar is deflected and allowed to oscillate from a static position $\alpha$ = 180° = $\pi$ from its rest position.

| *r* / m | *F* / N |
|---|---|
| 0.05 | 1.72 |
| 0.10 | 0.86 |
| 0.15 | 0.58 |
| 0.20 | 0.46 |
| 0.25 | 0.32 |
| 0.30 | 0.26 |

**Determination of moment of inertia $J_0$ for dumbbell bar without weights**

Period of oscillation $T_0$: 2460 ms

**Determination of moment of inertia *J* as a function of the distance of weights *r* from the axis of rotation**

Tab. 2: Period of oscillation *T* for a dumbbell bar with weights attached at a distance *r* from the axis of rotation

| *r* / m | *T* / ms |
|---|---|
| 0.05 | 2825 |
| 0.10 | 3663 |
| 0.15 | 4740 |
| 0.20 | 5926 |
| 0.25 | 7170 |
| 0.30 | 8440 |

**Determination of moment of inertia *J* for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder**

Tab. 3: Period of oscillation *T* for various sample bodies

| Sample bodies | *T* / ms |
|---|---|
| Disc | 1800 |
| Sphere | 1880 |
| Pan | 512 |
| Solid cylinder + pan | 917 |
| Hollow cylinder + pan | 1171 |

**Verification of Huygens-Steiner theorem**

Tab. 4: Period of oscillation *T* for oscillation of a wooden disc around points at various distances *a* from its centre of gravity.

| *a* / cm | *T* / ms |
|---|---|
| 0 | 2922 |
| 2 | 2960 |
| 4 | 3121 |
| 6 | 3327 |
| 8 | 3622 |
| 10 | 3948 |
| 12 | 4359 |
| 14 | 4748 |

# EVALUATION

**Determining torsional coefficient $D_r$ for coupling springs**

From equation (5), the following applies:

$$(6) \quad F = \alpha \cdot D_r \cdot \frac{1}{r} = C \cdot \frac{1}{r} \quad \text{where } C = \alpha \cdot D_r$$

- Plot the force measurements *F* from Table 1 against the inverse of the distances 1/*r* and draw a straight line through the measurement points.

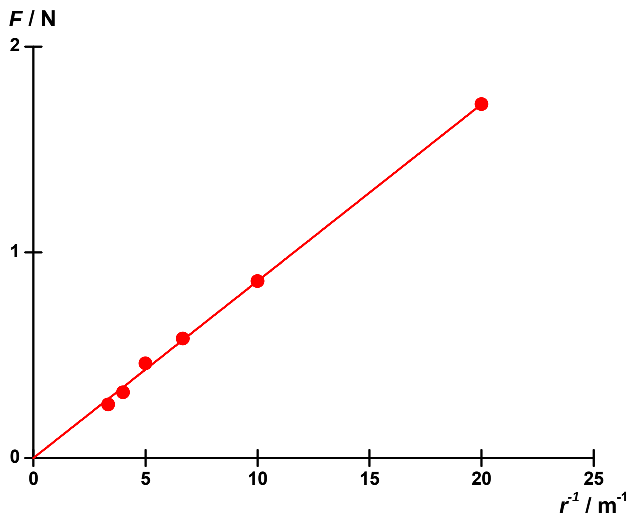


Fig. 2: Force *F* as a function of the inverse of the distance of the weights from the centre 1/*r*.

- Determine the torsional coefficient $D_r$ from the gradient *C* as indicated in equation (6):

$$(7) \quad C = \alpha \cdot D_r \Leftrightarrow D_r = \frac{C}{\alpha} = \frac{0.0860\ \text{Nm}}{\pi} = 0.0274\ \text{Nm}\ .$$

**Determination of moment of inertia $J_0$ for dumbbell bar without weights**

From equation (4), the moment of inertia for the dumbbell bar without weights is given by:

$$(8)\quad J_0 = 0.0274\,\text{Nm}\cdot\frac{(2.460\,\text{s})^2}{4\pi^2} = 4.20\cdot 10^{-3}\,\text{kg}\cdot\text{m}^2 .$$

**Determination of moment of inertia *J* as a function of the distance of weights *r* from the axis of rotation**

- Use equation (4) to calculate the moments of inertia *J* of the dumbbell bar with weights from the values in Table 2 and enter them into Table 5.
- Deterine the moments of inertia of the weights themselves $J_m$ from the following

  $$(9)\quad J_m = J - J_0$$

  Enter the results into Table 5

Tab. 5: Period *T*, moment of inertia *J* for dumbbell bar with weights plus moment of inertia $J_m$ for weights at various distances r from the axis of rotation.

| *r* / m | *T* / s | *J* / $10^{-3}$ kg·m² | $J_m$ / $10^{-3}$ kg·m² |
|---|---|---|---|
| 0.05 | 2.825 | 5.54 | 1.34 |
| 0.10 | 3.663 | 9.31 | 5.11 |
| 0.15 | 4.740 | 15.6 | 11.4 |
| 0.20 | 5.926 | 24.4 | 20.2 |
| 0.25 | 7.170 | 35.7 | 31.5 |
| 0.30 | 8.440 | 49.4 | 45.2 |

According to (2):

$$(10)\quad J_m = 2\cdot m\cdot r^2$$

- Plot the moments of inertia $J_m$ from Table 5 against the square of the distance $r^2$ and confirm the linear relationship expected from equation (10) (Fig. 3).

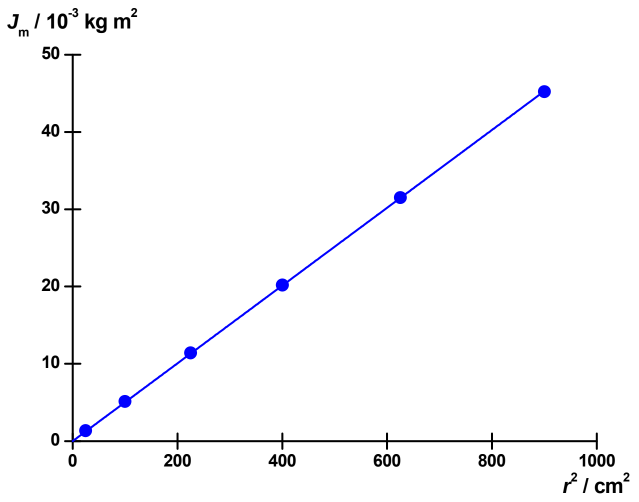


Fig. 3: Moment of inertia of weights $J_m$ as a function of the distance of the weights from the axis *r*.

**Determination of moment of inertia *J* for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder**

- Using equation (4), calculate the moments of inertia *J* for the various sample bodies and the measurements entered into Table 3 and enter the results into Table 6.
- In order to determine the moments of inertia of the solid cylinder and the hollow cylinder $J_V$ and $J_H$, in each case subtract the moment of inertia of the pan $J_T$ from the values of the moment of inertia for the solid cylinder + pan or the hollow cylinder + pan $J_{VT}$ and $J_{HT}$:

  $$(11)\quad \begin{array}{l} J_V = J_{VT} - J_T \\ J_H = J_{HT} - J_T \end{array} .$$

- Calculate the theoretical values for the moment of inertia $J_{th}$ with the help of the data in the appendix. Enter them into Table 6 and compare them with the values determined by measurement.

Tab. 6: Moments of inertia *J* for various sample bodies.

| Sample bodies | *T* / s | *J* / $10^{-3}$ kg·m² | $J_{th}$ / $10^{-3}$ kg·m² |
|---|---|---|---|
| Disc | 1.800 | 2.25 | $^1/_2$·m·$r^2$ = 2.57 |
| Sphere | 1.880 | 2.45 | $^2/_5$·m·$r^2$ = 2.54 |
| Pan | 0.512 | 0.18 | – |
| Solid cylinder + pan | 0.917 | 0.58 | – |
| Solid cylinder | – | 0.40 | $^1/_2$·m·$r^2$ = 0.43 |
| Hollow cylinder + pan | 1.171 | 0.95 | – |
| Hollow cylinder | – | 0.77 | m·$r^2$ = 0.86 |

The values determined by measurement are well in agreement with those calculated in accordance with the theory.

**Verification of Huygens-Steiner theorem**

- Determine the moments of inertia $J_a$ for various distances *a* from the measurements in Table 4 using equation (4) and enter these values into Table 7.

Tab. 7: Moment of inertia $J_a$ for a disc oscillating about various axes at distance *a* from its centre of gravity.

| *a* / cm | *T* / s | $J_a$ / $10^{-3}$ kg·m² |
|---|---|---|
| 0 | 2.922 | 5.93 |
| 2 | 2.960 | 6.08 |
| 4 | 3.121 | 6.76 |
| 6 | 3.327 | 7.68 |
| 8 | 3.622 | 9.11 |
| 10 | 3.948 | 10.8 |
| 12 | 4.359 | 13.2 |
| 14 | 4.748 | 15.6 |

- Gemäß dem Steiner'schen Satz gilt:

  (12) $J_a = J_0 + m \cdot a^2$ where $J_0 = J_a(a=0)$

- Plot $J_a - J_0$ against $a^2$, and confirm the linear relationship indicated in equation (12), thereby verifying the Huygens-Steiner theorem (Fig. 4).

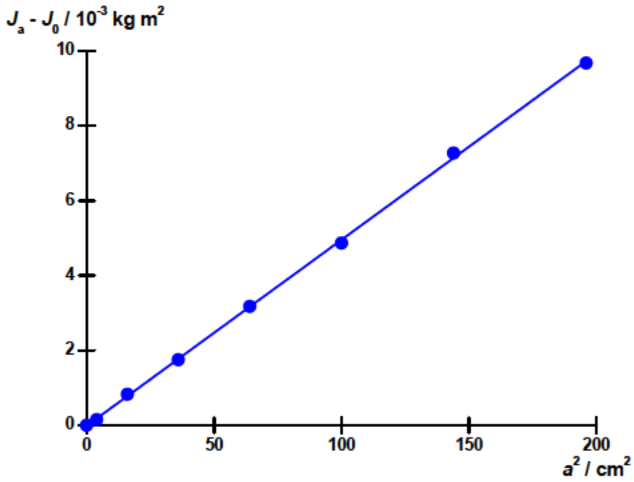


Fig. 4: Difference in moments of inertia $J_a - J_0$ of a disc as a function of the distance *a* of the axis of rotation from its centre of gravity.

## APPENDIX: TECHNICAL DATA

**Dumbbell bar**

| | |
|---|---|
| Length: | 620 mm |
| Weight: | approx. 135 g |
| Weights: | 260 g each |

**Disc**

| | |
|---|---|
| Diameter: | 320 mm |
| Weight: | approx 495 g |
| Boreholes: | 8 |
| Borehole spacing: | 20 mm |

**Wooden sphere**

| | |
|---|---|
| Diameter: | 146 mm |
| Weight: | approx. 1190 g |

**Wooden disc**

| | |
|---|---|
| Diameter: | 220 mm |
| Height: | 15 mm |
| Weight: | approx. 425 g |

**Mounting plate**

| | |
|---|---|
| Diameter: | 100 mm |
| Weight: | approx. 122 g |

**Solid cylinder (wood)**

| | |
|---|---|
| Diameter: | 90 mm |
| Height: | 90 mm |
| Masse | approx. 425 g |

**Hollow cylinder (metal)**

| | |
|---|---|
| External diameter: | 90 mm |
| Height: | 90 mm |
| Weight: | approx. 425 g |