PHYSICS EXPERIMENT

Adiabatic exponent of air

DETERMINATION OF ADIABATIC EXPONENT C_P/C_V FOR AIR USING RÜCHARDT'S TECHNIQUE.

- Measure the period of oscillation of an aluminium piston.
- Determine the equilibrium pressure in an enclosed volume of air.
- Determine the adiabatic exponent of air and compare it with the value quoted in tables.

UE2040200

03/15 UD

GENERAL PRINCIPLES

It is possible to determine the adiabatic exponent of air using a classic experiment devised by Rüchardt. The exponent is derived from the vertical oscillations of a piston which rests on top of a cushion of air inside a tube of constant cross-section and seals the air in from above. Deflecting the piston from its rest position causes the pressure of the volume of air to rise above or below atmospheric in such a way as to propel the piston back towards its rest position. This restoring force is proportional to the deflection from the rest position, meaning the oscillation of the piston is a simple harmonic motion.

Since no heat is transferred to the surroundings, the oscillations are associated with adiabatic changes in state. The following equation describes the relationship between pressure p and volume V of the enclosed air:

(1) $p \cdot V^{\gamma} = \text{const.}$

The adiabatic exponent γ is the ratio of the specific heat capacities at constant pressure C_p and at constant volume C_V:

(2) $\gamma = \frac{C_p}{C_V}$.

Equation (1) implies the following for pressure and volume changes Δp and ΔV

(3) $\Delta p + \gamma \cdot \frac{p}{V} \cdot \Delta V = 0$.

By substituting the internal cross-sectional area of the tube A into the equation, the change in pressure allows the restoring force ΔF to be deduced, while the deflection of the piston from its rest position Δs can be calculated from the change in volume.

The following therefore results:

(4) $\Delta F = -\gamma \cdot \frac{p}{V} \cdot A^2 \cdot \Delta s = 0$

This can then be converted to an equation of motion for the piston as follows:

(5) $m \cdot \frac{d^2 \Delta s}{dt^2} + \gamma \cdot \frac{p}{V} \cdot A^2 \cdot \Delta s = 0$.

m: Mass of piston

The solution to this classical equation of motion for a simple harmonic oscillator corresponds to oscillations with the following period:

(6) $T = 2\pi \sqrt{\frac{1}{\gamma} \cdot \frac{V}{p} \cdot \frac{m}{A^2}}$,

From this it is possible to calculate the adiabatic coefficient as long as the other variables are known.

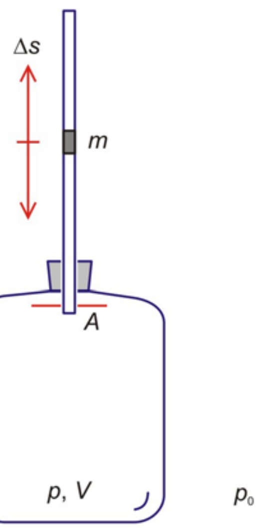

Fig. 1: Schematic representation of measurement set-up.

The experiment involves the use of a precision glass tube with a small cross-section *A* inserted vertically into the rubber stopper of a glass vessel with a large volume *V*. The piston is an aluminium cylinder of matching area and known mass *m* which can slide up and down the tube. The aluminium cylinder undergoes simple harmonic oscillation by bouncing on the cushion formed by the enclosed volume of air. The adiabatic exponent can be calculated from the period of oscillation of the aluminium cylinders.

Fig. 2: Measurement set-up.

LIST OF EQUIPMENT

1	Mariotte flask	1002894
1	Oscillation tube	1002895
1	Mechanical stopwatch, 15 min	1003369
1	Vacuum hand pump	1012856

The following apparatus is recommended in addition for the purpose of measuring the ambient air pressure, the internal diameter of the oscillation tube and the mass of the aluminium cylinder:

1	Aneroid barometer F	1010232
1	Callipers, 150 mm	1002601
1	Electronic scales, 200 g	1003433

PROCEDURE

- Determine the air pressure, the internal diameter of the tube in which the cylinder will oscillate, the mass of the aluminium cylinder itself and the volume of the Mariotte flask.
- Put one of the two conical rubber stoppers, fat end first, into the conical opening of the Mariotte flask and press on it gently. This prevents the aluminium cylinder from falling into the flask.
- The Mariotte flask should also be equipped with a rubber mat or similar to prevent any damage to the bottle itself or to the aluminium cylinder if it were to fall in.
- Set the tube up on top of the Mariotte flask, making sure it is vertical by clamping it to a stand if necessary.
- Connect one end of the long hose (850 mm, 6.5 mm internal diameter) from the equipment included with the vacuum hand pump to the 3-way cock of the Mariotte flask and close the 3-way cock.
- Clean the aluminium cylinder with a lint-free cloth and some cleaning solvent. With the 3-way cock still closed, insert the cylinder into the oscillation tube making sure that it does not get tilted, then let it fall. You should only touch the aluminium cylinder by the handle in order to stop it getting dirty.
- Use the mechanical stop watch to measure the time needed for five oscillations. Start measuring the time when the aluminium slows to a stop at its lowest point for the first time. Stop the measurement on the sixth occasion it reaches that point.
- Carefully open the 3-way cock so that the aluminium cylinder can slowly descend to the rubber stopper at the bottom of the oscillation tube.
- Connect the vacuum hand pump via a hose to the 3-way cock of the Mariotte flask. With the cock open, pump the aluminium cylinder back up the tube and take it out. Make sure that the cylinder does not fall out of the tube and get damaged.
- Take the aluminium cylinder out of the tube completely, thereby allowing the system to return to the ambient air pressure. Close the three-way cock again and disconnect the vacuum pump from the hose.
- Carry out nine more measurements.

Important note: The quality of these measurements strongly depends on the following conditions:

- The tube in which the oscillation takes place needs to be extremely clean. If necessary you should clean it with tissue.
- The aluminium cylinder also needs to be extremely clean. Even small amounts of dirt, such as grease from your skin, can cause serious friction. Therefore, you should clean the aluminium cylinder with a lint-free cloth and some cleaning solvent before every measurement.

- The slightest deformation of the aluminium cylinders (as a result of dropping it, for example) adversely affects the measurement.
- The tube in which the oscillations occur needs to be absolutely vertical.
- All the rubber stoppers have to be airtight.
- Measurement of time needs to be carried out very carefully, since the period of oscillation is squared in the equation for the measurement (8) (see sample measurements and evaluation).
- The internal diameter of the tube has to be measured extremely accurately, since its radius must be squared to find the cross-sectional area of the tube *A* and since that variable is itself squared in equation (8), the radius appears in the equation raised to the fourth power.

SAMPLE MEASUREMENTS AND EVALUATION

External air pressure p_0: 1018 mbar

Internal diameter d_i of the oscillation tube: 16 mm

Mass m of aluminium cylinder: 15.2 g

Volume V_0 of Mariotte flask: 10400 cm^3

Time T_5 for 5 periods of oscillation (10 measurements):

5.172 s	5.276 s	5.259 s	5.224 s	5.305 s
5.175 s	5.231 s	5.241 s	5.191 s	5.175 s

Average value for T_5 from 10 measurements: 5.225 s

Period of oscillation T: 1.045 s

The equilibrium pressure p results from the external pressure p_0 and the pressure the aluminium cylinder exerts on the enclosed air when at rest:

(7) $p = p_0 + \frac{m \cdot g}{A}$, g = acceleration due to gravity.

The equilibrium volume V corresponds to the volume V_0 of the Mariotte flask since the volume of the oscillation tube can be neglected.

From equation (6), the following is used to determine the adiabatic exponent:

(8) $\gamma = \left(\frac{2\pi}{T}\right)^2 \cdot \frac{m}{A^2} \cdot \frac{V}{p} = 1{,}39$.

The value measured is very well in agreement with the theoretical value γ =7/5 = 1.4 for a diatomic molecule with 3 degrees of translational freedom and 2 degrees of rotational freedom.