Impedance of a Capacitor in an AC Circuit

DETERMINE THE IMPEDANCE OF A CAPACITOR AS A FUNCTION OF CAPACITANCE AND FREQUENCY

- **Determine the amplitude and phase of capacitive impedance as a function of the capacitance.**
- **Determine the amplitude and phase of capacitive impedance as a function of the frequency.**

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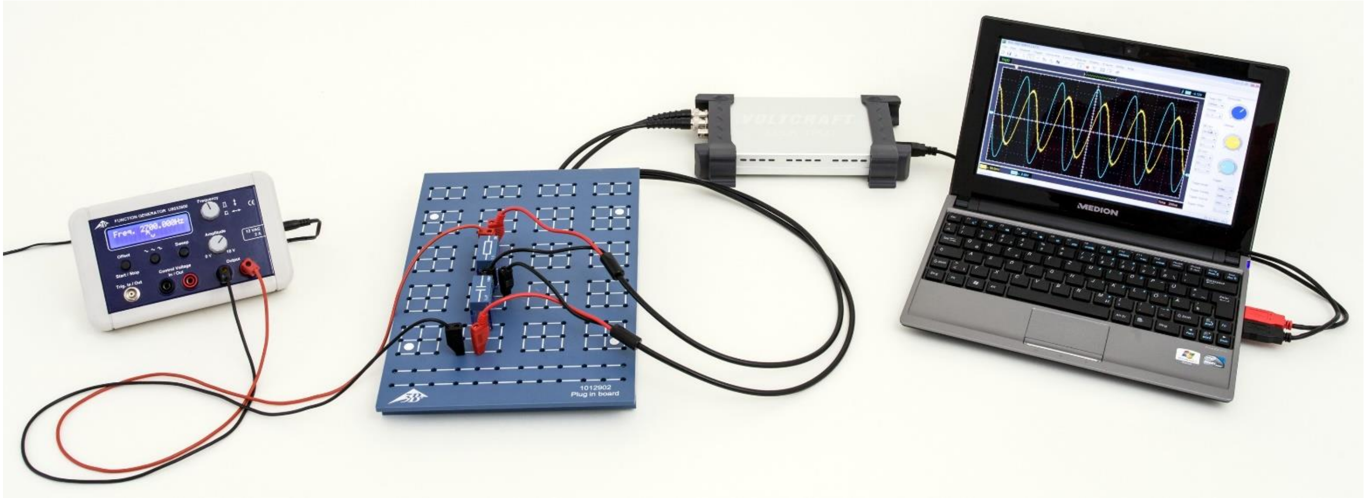

Fig. 1: Experiment set-up

GENERAL PRINCIPLES

Any change in voltage across a capacitor gives rise to a flow of current through the component. If an AC voltage is applied, alternating current will flow which is shifted in phase with respect to the voltage. In mathematical terms, the relationship can be expressed most easily if current, voltage and impedance are regarded as complex values, whereby the real components need to be considered.

The capacitor equation leads directly to the following:

(1) $$I = C \cdot \frac{\mathrm{d}U}{\mathrm{d}t}$$

I: Current, U: Voltage, C: Capacitance

Assume the following voltage is applied:

(2) $$U = U_0 \cdot \exp(i\omega t)$$

This gives rise to a current as follows:

(3) $$I = i \cdot \omega \cdot C \cdot U_0 \cdot \exp(i\omega t)$$

Capacitor C is then assigned the complex impedance

(4) $$X_C = \frac{U}{I} = \frac{1}{i \cdot \omega \cdot C} = \frac{1}{i \cdot 2\pi \cdot f \cdot C}$$

The real component of this is measurable, therefore

(5) $$U = U_0 \cdot \cos\omega t$$

(6) $$I = \omega \cdot C \cdot U_0 \cos\left(\omega t + \frac{\pi}{2}\right) = I_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

(7) $$X_C = \frac{U_0}{I_0} = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C}$$

In this experiment, a frequency generator supplies an alternating voltage with a frequency of up to 5 kHz. A dual-channel oscilloscope is used to record the voltage and current, so that the amplitude and phase of both can be determined. The current through the capacitor is related to the voltage drop across a resistor *R* with a value which is negligible in comparison to the impedance exhibited by the capacitor itself.

As an option, voltage and current can also be recorded using the VinciLab data logger and Coach 7 software with voltage sensors.

LIST OF EQUIPMENT

1	Plug-In Board for Components	1012902 (U33250)
1	Resistor 1 Ω, 2 W, P2W19	1012903 (U333011)
1	Resistor 10 Ω, 2 W, P2W19	1012904 (U333012)
3	Capacitor 1 µF, 100 V, P2W19	1012955 (U333063)
1	Capacitor 0.1 µF, 100 V, P2W19	1012953 (U333061)
1	Function Generator FG 100 @230V	1009957 (U8533600-230)
or		
1	Function Generator FG 100 @115V	1009956 (U8533600-115)
1	Set of 15 Experiment Leads, 1 mm²	1002840 (U13800)
1	PC Oscilloscope 2x25 MHz	1020857 (U11830)
2	HF Patch Cord, BNC/4 mm Plug	1002748 (U11257)
Optional		
1	VinciLab	1021477 (UCMA-001)
1	Coach 7, School Site License 5 Years	1021522 (UCMA-18500)
or		
1	Coach 7, University License 5 Years	1021524 (UCMA-185U)
2	Voltage Sensor 10 V, Differential	1021680 (UCMA-0210i)
1	Voltage Sensor 500 mV, Differential	1021681 (UCMA-BT32i)
1	Sensor Cable	1021514 (UCMA-BTsc1)

SET-UP AND EXPERIMENT PROCEDURE

- Set up the measuring equipment (Fig. 1) as shown in the circuit diagram (Fig. 2) with resistor R =1 Ω and a capacitor C = 1 µF.
- Connect the measuring lead for the recording of the voltage curve $U_R(t) = R \cdot I(t)$ across the resistor to channel CH1 and the measuring lead for recording the curve $U_C(t)$ across the capacitor to channel CH2 of the oscilloscope.

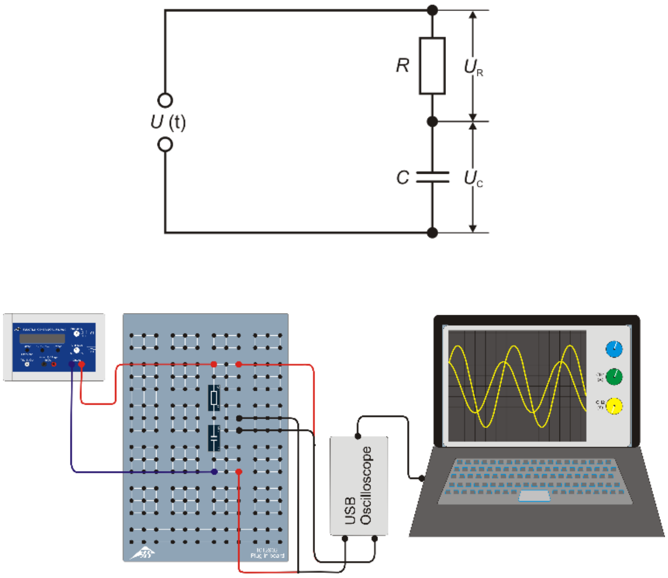

Fig. 2: Circuit diagram sketch (top) and set-up schematic (bottom).

- Set the following parameters on the PC oscilloscope:

Horizontal:	
Time base:	50 µs/div
Horizontal trigger position:	0.0 ns
Vertical:	
CH1:	
Voltage scale division:	50 mV/div DC
Zero position:	0.0 divs
CH2:	
Voltage scale division:	1 V/div DC
Zero position:	0.0 divs
Trigger:	
Single (not Alternate)	
Source:	CH2
Mode:	Edge
Edge:	Rising
Threshold:	0.000 mV
Trigger mode:	Auto

- It may be necessary to change the Time/div and Volts/div settings during the series of measurements to ensure the signal is optimally displayed.
- Set the frequency to f = 4000 Hz.
- Select a sinusoidal wave form on the function generator and adjust the amplitude of the input signal to U_0 = 4 V. Set the amplitude control in such a way that the maximum and minimum of the sinusoidal signal on channel CH2 of the oscilloscope are separated by four divisions (for a setting of 1 V/div).

Since the value of the resistor R is negligible in comparison to the capacitive impedance of the capacitor X_C at the frequencies being observed, the following formula is a good approximation for the situation $U_{C0} \approx U_0$ = 4 V.

Phase shift between current and voltage

- Observe and make a note of the relative positions of the voltage curves $U_C(t)$ and $U_R(t)$ across the capacitor and resistor.

How the capacitive impedance of the capacitor depends on the capacitance

- Using the 0.1 µF capacitor in both series and parallel circuits including the three 1 µF capacitors, set up circuits with the capacitance values listed in Table 1. For each setting read off the amplitudes U_{R0} from the scope and enter them into Table 1 as well.

How capacitive impedance depends on frequency

- Use the 1 µF capacitor and 10 Ω resistor for the measurements
- Set up the frequencies listed in Table 2 on the function generator one by one, read off amplitudes U_{R0} from the oscilloscope and enter them into Table 2 as well.

SAMPLE MEASUREMENT AND EVALUATION

Phase shift between current and voltage

The current signal is shifted by a quarter of the period to the right with respect to the voltage signal (Fig. 3).

The current through the capacitor leads the voltage across it by 90° since the current charging the capacitor (positive sign) and the current when the capacitor is discharging (negative sign) are at their maximum levels when the voltage crosses the zero axis.

Tab. 1: How capacitive impedance depends on capacitance, f = 4000 Hz, R = 1 Ω, U_0 = 4 V.

C µF	U_{R0} mV	$1/C$ 1/µF	$I_0=U_{R0}/R$ mA	$X_C=U_0/I_0$ Ω
0.10	9.3	10.0	9.3	430.1
0.33	32.1	3.0	32.1	124.6
0.50	51.1	2.0	51.1	78.3
0.67	67.8	1.5	67.8	59.0
1.00	101.7	1.0	101.7	39.3
2.00	204.3	0.5	204.3	19.6

Tab. 2: How capacitive impedance depends on frequency C = 1 µF, R = 10 Ω, U_0 = 4 V.

f Hz	U_{R0} mV	$1/f$ 1/kHz	$I_0=U_{R0}/R$ mA	$X_C=U_0/I_0$ Ω
200	50	5.00	5	800
300	78	3.33	8	513
500	127	2.00	13	315
1000	255	1.00	26	157
2000	493	0.50	49	81
3000	733	0.33	73	55
4000	993	0.25	99	40
5000	1203	0.20	120	33

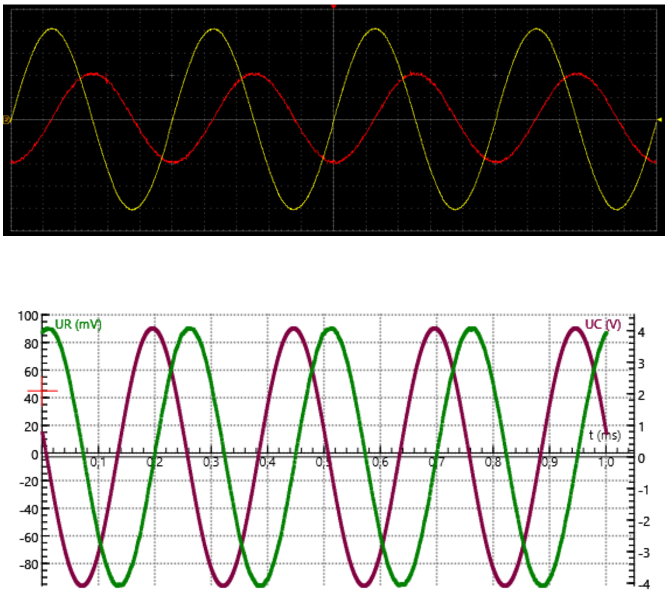

Fig. 3: Capacitor in AC circuit: trace of voltage and current. Top: Recording using PC oscilloscope (current: red, voltage: yellow). Bottom: Recording using VinciLab/Coach7 (current: green, voltage: violet).

How capacitive impedance depends on capacitance and frequency

- Plot the capacitive impedance values X_C against the inverse of the capacitance (Table 1, Fig. 4) and the frequency (Table 2, Fig. 5).

As per equation (4) the capacitive impedance X_C is proportional to the inverse of the frequency f and the inverse of the capacitance C. In the relevant graphs, the measurements therefore lie along a straight line through the origin within the measurement tolerances.

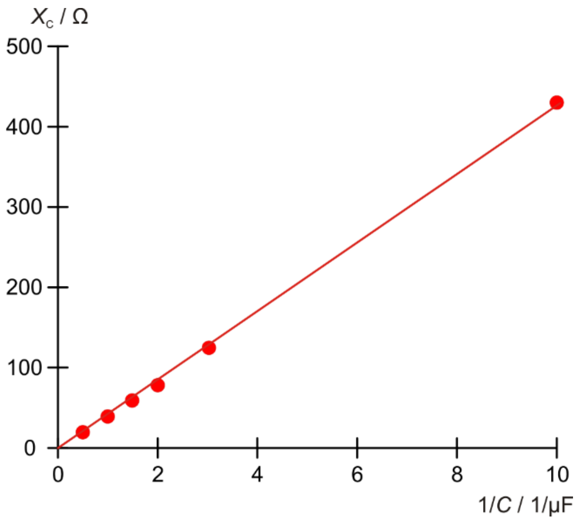

Fig. 4: Capacitive impedance X_C as a function of the inverse of the capacitance C.

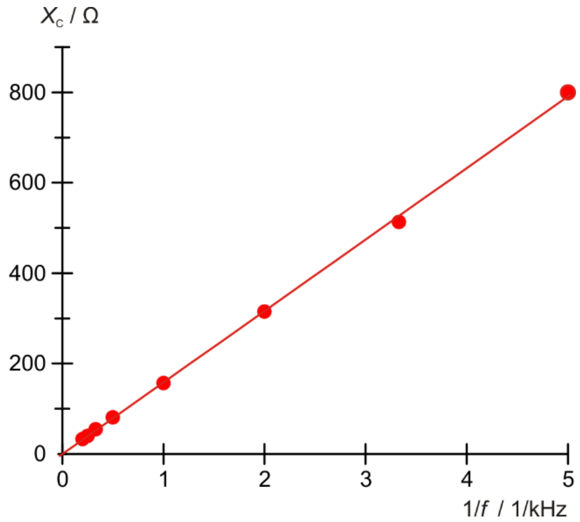

Fig. 5: Capacitive impedance X_C as a function of the inverse of the frequency *f*.